

Neither black-holes nor regular solitons: a no-go theorem

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Abstract

By studying the BPS equations for electrostatic and spherically symmetric configurations in $N = 2$, $d = 5$ gauged supergravity with vector multiplets and hypermultiplets coupled, we demonstrate that no regular supersymmetric black-hole solutions of this kind exist. Furthermore, we demonstrate that it is not possible to construct supersymmetric regular solitons that have the above symmetries. As a consequence the scalar flow associated to the BPS solutions is always *unbounded*.

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1 Introduction and result

Since the advent of the duality between String theory (or, for energies much smaller than the string scale, Supergravity) on anti-de Sitter (AdS) spaces and supersymmetric conformal field theory residing on the conformal boundary of AdS [1, 2, 3, 4], there has been a renewed interest in the study of gauged supergravity in various dimensions. A preeminent role has been played in this contest by special types of solutions like domain walls, black strings and black holes, that admit an interesting holographic interpretations.

Naturally, a considerable attention has been taken by the five-dimensional supergravity, especially in its minimal supersymmetric formulation. This theory has eight supercharges ($N = 2$ in the four dimensional language) and shares a lot of properties with the theories in four and six dimensions with same amount of supersymmetry.

The five-dimensional theory is interesting because provides a natural effective set-up to study the extension of the correspondence beyond the conformal limit and in presence of the less supersymmetry. It also favors the construction of phenomenological model with large extra dimensions like the Randall-Sundrum scenarios [5, 6] and it is crucial in any attempt to accommodate them with string theory [7].

Although a lot of works have been devoted to construct special supersymmetric solutions or/and to classify the possible ones, most of them remain unknown, especially for the full-fledge $N = 2$, $d = 5$ gauged supergravity coupled to matter. Indeed, even though the program of classifying the theory has been carried out almost¹ to completion in various steps [8, 9, 10, 11, 12, 13, 14], for most of the solutions we have only an implicit construction and the full implication of the classification, at least when hypermultiplets are coupled, remains to be reveal.

Also for the few classes of BPS solutions that have been studied starting by clever chosen ansatz, it is difficult to recognize all the consequences that the reach geometric structure of

¹ $N = 2$, $d = 5$ gauged supergravity with tensor multiplets coupled has not been considered yet.

the theory has. A nice example is given by domain walls. Surprisingly, it was shown only quite recently [15], under the inspiration of [16], that supersymmetric domain walls can be “curved” [17, 18, 19, 20, 21] only when hypermultiplets are not trivially coupled.

Nevertheless, the domain walls, together with some of their deformations [22], are the only configurations studied in full generality for all the possible multiplets coupling.

This is not true for black holes, which have been considered in full extent only in presence of vector multiplets [23, 24, 25, 10, 26, 27, 28, 29, 30]. More in general, not a lot is known about charged solutions, especially in presence of charged matter (hypermultiplet) [31, 32, 33].

In this work we try to partially cover this gap. We consider all 1/2 BPS electrostatic and spherically symmetric configurations of the most general $N = 2$, $d = 5$ supergravity gauged by an abelian gauge group. In our parametrization, they are described by the metric

$$ds^2 = -e^{2v} dt^2 + e^{2w} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\xi^2), \quad (1.1)$$

where r represents “physical” radius. The choice of the ansatz is completed by taking the gauge field as the electrostatic potential. The scalars and all the other fields depends only on r , in agreement with the $SO(4)$ symmetry. We analyze the BPS equations for these configurations previously derived in [31, 32].

Some of the solutions are already well known. For example, this class contains the superstar solutions [23], singular black hole configurations that can be deformed to non BPS regular black holes simply by the introduction of a mass term [24]. These configuration are obtained in a particular vector multiplet model, called “STU” that is a consistent truncation of $N = 8$, $d = 5$ gauged supergravity. As a consequence, the superstars correspond to the consistent reduction of type IIB solutions on S^5 and they can be uplifted back [34] to ten-dimensions. The superstars have received a lot of attention for their interpretation in terms of Giant graviton excitations [35]. Finally, in [36], it has been shown that the superstars arise as singular limit of regular 1/2 BPS “AdS bubbles” (see also [37]). Recently, these singular black hole configurations have been investigated in the Fake supergravity formalism [38]. Furthermore, the analogous of the superstar solutions in the minimal gauged five-dimensional supergravity can also be interpreted as a solution of IIB, reduced this time on $Y^{(p,q)}$ [39]. As from the ten-dimensional point of view this solution preserves four supercharges, its non BPS deformations is dual to the quark-gluon plasma of the $N = 1$, $d = 4$ quiver gauge theory.

A different example of solutions (which is not asymptotically AdS_5) in the family of (1.1), whose origin in string/M-theory is known, is given by the “axionic-shift” considered in [40] and further discussed in [31].

Here, we are interested in the understanding of the general properties of the solutions (1.1). In practise, our aim is draw the identikit of the possible configurations and determine their qualitative behavior by the analysis of the BPS equations. Due to the symmetry of the problem this can be done without relaying on the specific features of a particular model but only on the geometric properties of the scalar manifold. By using such properties we are able to demonstrate that:

- there are no supersymmetric (extreme) Reissner-Nordström in $N = 2$ $d = 5$ gauged supergravity with abelian gauging;
- no other regular solution than the supersymmetric vacuum exists, no matter on which matter multiplets are coupled.

These results are obtained in various steps and are based on the analysis of the BPS equations previously derived in [32]. We note that when only vector multiplets are present the existence of regular spherically symmetric BPS black holes has been already ruled out in² [41, 42].

By first, we observe that a necessary condition for a solution to be regular is that the corresponding superpotential W is a regular and limited function of the scalar manifold. As the BPS equations impose that W is a monotonic decreasing function of r , $W' < 0$, this is equivalent³ to ask that $W_0 \equiv W(r = 0)$ is finite. As explained in section 3.2 the finiteness of W is not only *necessary* but also *sufficient*.

Armed of this property, we are able to derive our no-go theorems via *reductio ad absurdum*. Indeed, as a first consequence of the assumption of W finite, we show that regular solutions can only end at the $r = 0$.

Then, we prove that such solutions cannot be black holes because g_{tt} and g_{rr} are constant at the origin.

By considering the BPS equations for the scalars, we argue that the regularity imposes $W' = 0$ at $r = 0$. This automatically implies that the vector multiplet scalars should reach an attractor point of W , which means $\partial_x W|_{r=0} = 0$. Such an attractor point should be infrared attractive. This gives immediately a contradiction, due to the famous No-go theorem [44, 45] for very special real geometry, hence regular solutions with only vector multiplets non trivially coupled are excluded.

By the analysis of the hypermultiplet sector, we first show that, in order to be regular and to satisfy e.o.m., a supersymmetric configuration must approach a fixed point of W , i.e. $\partial_X W|_{r=0} = 0$. This is irrespective of the presence of vector multiplets. Then, by considering hypermultiplets only, we demonstrate that no flow can end at a fixed point for $r = 0$: we conclude that the only BPS solution (fully) regular in the origin is the maximally supersymmetric AdS vacuum. Regarding the generic case, we show that, in opposition to what happens for the BPS domain wall flows [7] (which present a lot of similarity with the ones under study), the mixing terms in the superpotential are not sufficient to give an infrared fixed-point⁴, i.e. attractive for $r \rightarrow 0$. This is essentially due to the fact that we are studying charged configurations: because of the coupling with the electric field, the hyperscalars (the scalars of the hypermultiplets) get attracted to their fixed-point values quicker than the vectorscalars (the scalars of the vector multiplets) in a way that the behavior of the latter is not affected. We can thus conclude that the superpotential cannot be bounded

²The near-horizon geometry analysis of [41, 42] has been recently compared with the global properties imposed by the attractor mechanism in [43].

³The superpotential without loss of generality may be taken positive. If, in addition to the regularity in the bulk, we require a regular behavior at the radial infinity, the superpotential must be also bounded from below, $W_0 \geq W \geq W_\infty$, where $W_\infty \equiv W(r = \infty) > 0$.

⁴This terminology is not fully appropriate in presence of hypermultiplets.

and no regular BPS solution different from the supersymmetric AdS vacuum exists.⁵ As a consequence, all the solutions that are asymptotically AdS at large r display a superstar-like behavior, irrespectively of the model considered.

2 A preliminary analysis of the BPS equation

The class of configurations that we are going to study are 1/2 BPS solutions of the $N = 2$, $d = 5$ gauged supergravity, charged under an abelian gauge group. These are bosonic solutions of the e.o.m.⁶ obtained by solving the first order BPS equations (2.10-2.15). These follow from the preservation of half of the supersymmetry. In this section we present such equations and we define all the quantities that are involved in the derivation of the no-go theorems of section 3. How the BPS equations have been obtained in [32], as well as the most important features of the supergravity theory, is reviewed in the appendix A.

The field content of most general $N = 2$, $d = 5$ gauged supergravity with an abelian gauged group is given by:

- the supergravity multiplet, containing the *graviton* e_μ^a , two *gravitini* $\psi_\mu^{\alpha i}$ and the *graviphoton* A_μ^0 ;
- n_H hypermultiplets, containing the *hyperini* ζ^A with $A = 1, 2, \dots, 2n_H$, and the real scalars q^X with $X = 1, 2, \dots, 4n_H$ which define a Quaternionic-Kähler manifold with metric g_{XY} ;
- n_V vector multiplets, containing the *gaugini* λ^{ia} , $a = 1, \dots, n_V$ of spin $\frac{1}{2}$, the real scalars ϕ^x , $x = 1, \dots, n_V$, which define a very special manifold with the metric g_{xy} and n_V *gauge vectors* A_μ^I , $I = 1 \dots, n_V$. Usually the graviphoton is included by taking $I = 0 \dots n_V$. At the same time, the very special manifold can be parametrized in terms of $n_V + 1$ coordinates $h^I(\phi)$, constrained by $C_{IJK}h^Ih^Jh^K = 1$.

The corresponding Lagrangian for the bosonic field is [47, 48]

$$\begin{aligned} \mathcal{L}_{BOS} = & \frac{1}{2}e\{R - \frac{1}{2}a_{IJ}F_{\mu\nu}^IF_{\mu\nu}^{J\mu\nu} - g_{XY}D_\mu q^XD^\mu q^Y - g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y - 2g^2\mathcal{V}(q, \phi)\} \\ & + \frac{1}{6\sqrt{6}}\epsilon^{\mu\nu\rho\sigma\tau}C_{IJK}F_{\mu\nu}^IF_{\rho\sigma}^JA_\tau^K. \end{aligned} \quad (2.1)$$

The gauge covariant derivative of the hypermultiplet scalars that are charged under the gauged group is

$$D_\mu q^X = \partial_\mu q^X + gA_\mu^IK_I^X(q),$$

⁵It is intriguing to conjecture that it might exist some link between this result and the absence of Kaluza-Klein AdS_5 solitons proved in [46].

⁶This has been explicitly proved in [32] for configurations under study. A general argument can be found in [14].

where $K_I^X(q)$ are Killing vectors on the quaternionic manifold. $\mathcal{V}(q, \phi)$ is the scalar potential, as given in Appendix A. A crucial property of the quaternionic geometry is that any isometry K_I^X is associated to a $SU(2)$ triplets of Killing prepotentials $P_I^r(q)$ ($r = 1, 2, 3$) via

$$D_X P_I^r = R_{XY}^r K_I^Y, \quad \Leftrightarrow \quad \begin{cases} K_I^Y = -\frac{4}{3} R^{YX} D_X P_I^r \\ D_X P_I^r = -\varepsilon^{rst} R_{XY}^s D^Y P_I^t, \end{cases} \quad (2.2)$$

where D_X denotes the $SU(2)$ covariant derivative, which contains an $SU(2)$ connection ω_X^r with curvature R_{XY}^r :

$$D_X P^r = \partial_X P^r + 2 \varepsilon^{rst} \omega_X^s P^t, \quad R_{XY}^r = 2 \partial_{[X} \omega_{Y]}^r + 2 \varepsilon^{rst} \omega_X^s \omega_Y^t. \quad (2.3)$$

The existence of the prepotential, which in the mathematical literature is known as moment map (see [49] and references therein), is essential to guarantee the supersymmetry invariance of the gauged theory. Furthermore it determines the BPS scalar equations, as we will see.

We can define the *dressed* prepotential P^r and the related Killing vector K^X as the linear combination $P^r \equiv P_I^r h^I$ and $K^X \equiv K_I^X h^I$. Finally, by decomposing P^r in its modulus and phase we have

$$P^r \equiv \sqrt{\frac{3}{2}} W Q^r, \quad Q^r Q^r = 1. \quad (2.4)$$

The scalar function W is known as *superpotential*. Let us remark that the prepotential $P^r(q, \phi)$, and as a consequence the superpotential $W(q, \phi)$, is always well-defined and smooth within the domain of the scalar manifold [49] (see also [50]).

Now, we are ready to define the ansatz for the solutions of interest. Indeed, the most general electrostatic and spherically symmetric configuration (up to gauge choice) can be expressed in terms of the metric

$$ds^2 = -e^{2v} dt^2 + e^{2w} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\xi^2), \quad (2.5)$$

and of the electrostatic potentials⁷

$$A_t^I = \sqrt{\frac{3}{2}} \Lambda e^v h^I(\phi). \quad (2.6)$$

In accordance with the symmetry, the functions v , w , Λ and the scalars can depend only on r . The requirement that half of the supersymmetry is preserved translates into the projector condition

$$\left(-i\Lambda\gamma_0\delta_i^j \pm \sqrt{1 - \Lambda^2} Q^r (\sigma_r)_i{}^j \gamma_1 \right) \epsilon_j = \epsilon_i. \quad (2.7)$$

A direct consequence of (2.7) is that $\Lambda^2 \leq 1$. It is a little more involved to show that

$$\partial_x Q^r = 0. \quad (2.8)$$

⁷With respect to [32], here we are interested in the generic case $\mu = l^x = 0$. The presence of a non trivial l^x is irrelevant for our discussion, as the scalar fields and the metric are independent of l^x .

As observed in [32], the same condition appears in study of BPS flat domain walls [7]. It has various consequence. From the point of view of the gauging, i.e. of the choice of the Killing vector K^X , it can be seen as a constraint as $\partial_x Q^r = 0$ is not satisfied *everywhere* for a generic linear combination of quaternionic isometries.

On other hands the above condition implies that the BPS solutions depend explicitly on the superpotential W *only* and not on the full prepotential P^r . A first hint of this fact can be deduces by the for the form of scalar potential, as it reduces to

$$\mathcal{V} = -6W^2 + \frac{9}{2}g^{\Lambda\Sigma}\partial_\Lambda W\partial_\Sigma W, \quad (2.9)$$

where the indices run over all the scalars, $\Lambda, \Sigma = 1, \dots, n_V + 4n_H$. The same it happens for the scalar flow. The complete set of independent BPS equations derived in [32] then is:

$$\partial_x Q^r = 0, \quad (2.10)$$

$$1 = \Lambda^2 e^{-2w} \left[1 + \frac{r}{2} \left(v' + \frac{\Lambda'}{\Lambda} \right) \right]^2, \quad (2.11)$$

$$e^v = \frac{r}{r_0 \sqrt{1 - \Lambda^2}}, \quad (2.12)$$

$$\sqrt{1 - \Lambda^2} = \mp g e^w r W, \quad (2.13)$$

$$q'^Z = \pm 3g e^w \sqrt{1 - \Lambda^2} \partial^Z W, \quad (2.14)$$

$$\phi'^x = \pm 3g e^w \frac{1}{\sqrt{1 - \Lambda^2}} \partial^x W. \quad (2.15)$$

This tells us that W is the only quantity to be investigated in order to determine the behavior of the BPS solutions. We will see that qualitatively such behavior is not going to depend too much on the details of the model.

We conclude this review of the BPS equations by noticing that the differential equation for the electrostatic potential (2.11) can be expressed in more clear way. By eliminating e^w due to (2.13) and defining

$$z = 2 \frac{1 - \Lambda^2}{\Lambda^2} \Rightarrow \Lambda^2 = \frac{1}{\frac{z}{2} + 1}, \quad (2.16)$$

(2.11) becomes

$$\dot{z} = 6z - 2z^2 f^2(y), \quad (2.17)$$

where $\dot{z} = \partial_y z$ with $y = \ln r$ and $f \equiv \frac{1}{grW}$. Treating W as a function of y , the equation (2.17) can be formally integrated as

$$z(y) = \frac{e^{6y}}{2 \int dy e^{6y} f^2}, \quad (2.18)$$

or equivalently

$$z(r) = \frac{r^6}{2 \int dr \frac{r^3}{(gW)^2}}. \quad (2.19)$$

The above equation implies for the metric

$$e^v = \frac{r}{r_0} \sqrt{\frac{z+2}{z}}, \quad (2.20)$$

$$e^w = \sqrt{\frac{z}{z+2}} f^2. \quad (2.21)$$

The “formal” integration of W can be made explicit in presence of vector multiplets only (or equivalently with the hypermultiplet scalars stabilized at constant vevs) because their BPS equation (2.15) can be made independent of z (Λ):

$$\dot{\phi}^x = -3g^{xy}\partial_x \ln W. \quad (2.22)$$

On the contrary for the hyperscalars we have

$$\dot{q}^X = -3\frac{z}{z+2}g^{XY}\partial_Y \ln W. \quad (2.23)$$

The different form of the above equations explains why is relatively easy to find superstar solutions analytically (see [23]) while it can be done only for constant Λ in presence of hypermultiplets [31]. The above equations will be deeply analyzed in the following sections. As a last comment, we note that the BPS solutions do not depend on the sign of W , which is related to the \pm signs in the equations (2.10-2.15). Without loss of generality W will be taken positive in the following.

3 Two no-go theorems

3.1 No BPS black holes

By definition, z must be positive. This implies that the integration constant in the denominator of

$$z(r) = \frac{r^6}{2 \int_0^r dr' \frac{r'^3}{(gW(r'))^2} + c}, \quad (3.1)$$

must be positive or the solution will be well-defined only for open interval $r > r^*$. If we are interested in the existence of black-hole solutions we need that $e^v \rightarrow 0$ as it approaches the horizon. From the equation (2.12) it is clear that this can occur only at the origin $r = 0$, then must be $c \geq 0$. Another crucial requirement for the existence of a regular horizon and then of a black hole is that the scalars have to approach a non singular fixed point (i.e. within the domain of the scalar manifold).

This is equivalent to ask that the superpotential admits a well-defined and *finite* value on the horizon, $\lim_{r \rightarrow 0} W = W_0$. Under this condition we are able to determine the behavior of z , and consequently of the metric for $r \approx 0$. Indeed, its behavior depends dramatically on the value of the integration constant c . For $c \neq 0$, $z \approx r^6$ then e^v is diverging in $r = 0$ as

$1/r^2$: the solution has a naked singularity. For $c = 0$ instead, the asymptotic behavior of z is

$$z \approx 2(gW_0r)^2, \quad (3.2)$$

and

$$e^v \approx \frac{1}{gW_0r_0}, \quad e^w = \frac{1}{gW_0r_0}e^{-v} \approx 1. \quad (3.3)$$

This is sufficient to conclude that NO SUPERSYMMETRY BLACK HOLE EXISTS with an abelian gauging (the non abelian case is not discussed here and the question remains open). Let us now discuss the existence of a regular solitons.

We note that $c = 0$ is a necessary condition in order to find regular solutions. Indeed, studying the case of c negative, which has been left apart, it can be shown that the solution encounters a curvature singularity for $r \rightarrow r^*$. This can be easily understood considering the solution for constant $W = W_0$, that corresponds to take an asymptotic limit of a generic regular solution. From (3.1) it follows:

$$z = 2(gW_0r)^2 \frac{1}{1 - (\frac{r^*}{r})^4}, \quad r^{*4} = -2c > 0, \quad (3.4)$$

and, using (2.21-2.21),

$$e^v = \frac{1}{gW_0r_0} \sqrt{(gW_0r)^2 + 1 - (\frac{r^*}{r})^4}, \quad (3.5)$$

$$e^w = \frac{1}{\sqrt{(gW_0r)^2 + 1 - (\frac{r^*}{r})^4}}, \quad (3.6)$$

$$A_t = \sqrt{\frac{3}{2}} \frac{r}{z r_0} = \frac{1}{gW_0r_0} \sqrt{\frac{3}{2} (1 - (\frac{r^*}{r})^4)}. \quad (3.7)$$

Although the metric and the electric potential are regular their derivatives are not: for example the electric field diverges as $(r - r^*)^{-2}$.

On the contrary, taking the limit $r^* \rightarrow 0$ we recover the $c = 0$ case and the above solution approach asymptotically the maximally supersymmetric AdS vacuum in spherical coordinates:

$$z = 2(gW_0r)^2, \quad (3.8)$$

$$e^v = \frac{1}{gW_0r_0} \sqrt{(gW_0r)^2 + 1}, \quad (3.9)$$

$$e^w = \frac{1}{\sqrt{(gW_0r)^2 + 1}}, \quad (3.10)$$

$$A_t = \sqrt{\frac{3}{2}} \frac{1}{gW_0r_0}. \quad (3.11)$$

Hence, we conclude that only the solutions with $c = 0$ can be regular at⁸ $r = 0$. In next session we will show that the only supersymmetric solution satisfying the hypothesis of a regular flow, i.e. $\exists \lim_{r \rightarrow 0} W = W_0$ *finite*, is AdS vacuum itself.

⁸The integration constant c does not play any role for $r \rightarrow \infty$.

3.2 Or constant or unbounded

In what follows we demonstrate that the assumption of a regular and *bounded* flow for the superpotential W automatically gives that W must be constant. Hence, in any model of the $N = 2$, $d = 5$ gauged supergravity with abelian gauging the superpotential associated to a supersymmetric soliton *diverges* in the infrared interior, pointing out that the gauged supergravity, viewed as an effective theory, breaks down in this limit. In order to explain more clearly the various step of our proof, we first proceed by analyzing what happens when hypermultiplets and vector multiplets are considered separately, and afterwards we discuss the generic case.

Before doing so, let us focus on the scalar equation (2.22) and (2.23). As argued in the previous section, in order to avoid a “conical singularity” in the scalar profile their derivative must be zero at the origin. This automatically implies, from eq.(2.22), that at $r = 0$ the flow has to reach a stationary point of W with respect the vector multiplet scalars, $\partial_x W = 0$. At this stage the analogous condition for the hypermultiplet scalars $\partial_X W = 0$ is not necessary because of the dependence on z : the factor $\frac{z}{z+2}|_{r \approx 0} \approx \frac{z}{2} \approx (W_0 r)^2$.

Just vectors

When only the vector multiplets are non trivially coupled the problem simplifies greatly and the observation above together with the properties of special geometry (the main formulas are given in the appendix A) is sufficient to rule out the existence of regular solution. In this case the superpotential W can be written as⁹ $W = W_I h^I(\phi)$, for some constant vector W_I . This implies that the fixed-point condition is $\partial_x W = -\sqrt{\frac{2}{3}} W_I h_x^I = 0$, or equivalently $h^I(\phi^*) = W^I/W$. Moreover, for the Hessian of W at $\phi = \phi^*$ it gives

$$\partial^y \partial_x W|_{\phi=\phi^*} = W_I \partial^y (-\sqrt{\frac{2}{3}} h_x^I)|_{\phi=\phi^*} = -\sqrt{\frac{2}{3}} W h_I(\phi^*) (\partial^y h_x^I)|_{\phi=\phi^*} = \frac{2}{3} W \delta_x^y. \quad (3.12)$$

This is essentially the no-go theorem [44, 45] derived studying the properties of domain wall solutions coupled to vector multiplets (for a compact review of supersymmetric domain walls in $N = 2$ supergravity in five dimensions see [15]). Indeed, the above relations imply that there is a unique fixed-point at $\phi = \phi^*$ and, by expanding the scalar equation (2.22) around it, one gets

$$r \partial_r \phi^x \approx -2(\phi - \phi^*)^x. \quad (3.13)$$

This tells us that the flow can reach ϕ^* only when r is going to infinity. The fixed-point is so called ultraviolet attractive due to the holographic interpretation of scalar equation as the β function of the dual theory. Indeed here r plays the role of the energy scale (as the scale factor does in the domain wall configurations).

⁹For an abelian gauging, as follows from the consistency condition (A.8), the prepotential P_I^r must not only constant but also parallel. As a consequence the condition for supersymmetry $\partial_x Q^r = 0$ is automatically satisfied.

We can thus conclude that the flow is not bounded, W diverges at r and the supersymmetric solutions are not regular. The solutions that are asymptotic to AdS_5 for large r are all superstar-like or AdS_5 itself.

Just hypers

As a preliminary step of our analysis, we show that the regularity of the solution forces the condition $\partial_X W = 0$ when $q'^X = 0 = W'$, hence for $r = 0$. This can be done in two ways. For example it can be shown by direct calculation, by expanding W around $r = 0$ and imposing that the e.o.m are satisfied at the origin.¹⁰ Indeed from (2.23) and (3.2) it follows

$$W|_{r \approx 0} = W_0 - \frac{3}{2}W_0(\partial^X W \partial_X W)r^2 + O[r]^3. \quad (3.14)$$

This expression can be used to compute z , e^v and e^w to the next order

$$z = 2W_0^2 r^2 (1 - 4(\partial^X W \partial_X W)r^2) + O[r]^5, \quad (3.15)$$

$$e^{2v} = \frac{1}{r_0^2 W_0^2} (1 + (W_0^2 + 2(\partial^X W \partial_X W))r^2) + O[r]^3, \quad (3.16)$$

$$e^{2w} = 1 - 3(W_0 - \partial^X W \partial_X W)r^2 + O[r]^3. \quad (3.17)$$

As a consequence, it can be shown the Einstein equations become equivalent to $\partial^X W \partial_X W = 0$. We note that the above result holds also in presence of vector multiplets, because they do not contribute to W'' and to the Einstein equations at the leading order at $r = 0$.

One can reach the same conclusion by arguing that, as observed in previous section, the regular solution must be asymptotic to AdS_5 for $r \rightarrow 0$. As the flow is supersymmetric, it must be $\partial_X W = 0$ because the AdS vacuum cannot preserve only a fraction of supersymmetry, hence it must be maximally supersymmetric. Practically the same argument applies for r going to infinity: the scalars (in this regime hyperscalars and vectorscalars behave in the same way) can be consistently attracted to a fixed-point if and only if the derivative of the superpotential with respect to the scalars is zero.

What we have just shown allows us to formulate our question in a way that is easier to answer. By introducing the parameter λ defined as $d\lambda = \frac{z}{r(z+2)} dr$, we are left to question the regularity of the flow

$$\frac{dq^X}{d\lambda} = -3\partial^X \log W. \quad (3.18)$$

Again, the problem seems equivalent to the analysis of renormalization group flow associated to the flat domain wall solutions studied in [7, 2.44]. In our contest, a supersymmetry solutions will be regular *if and only* the quaternionic scalars are attracted by a fixed-point for $r \rightarrow 0$. It is easy to conclude that this cannot be the case as λ is finite at $r = 0$. Indeed by expanding linearly around a fixed point q^* and integrating by part one gets that $\log |q - q^*| \propto \lambda$. This leads to a contradiction because the l.h.s. diverges at the fixed-point.

¹⁰The equations of motion are a consequence of the BPS equations when the scalars are not constant. Only if there is enhancement of supersymmetry this relation can be extended to hold also at the fixed point.

Hypers + vectors: the generic case

Now we are ready to discuss the generic case. The final result will be that the mixing terms arising due to the presence of both non trivial matter couplings do not help to escape the no-go theorem derived in the previous sections. The reason why this happens resides again in the factor $\xi(r) \equiv \frac{z}{z+2}$ in the hyperscalar equation (2.23). Its presence makes the analysis of the flow, and in particular of the properties of the fixed-points, strongly dependent on the two “regimes”: $r \approx \infty$ and $r \approx 0$. In the former case the analysis performed in [7] for the flat domain solutions applies. Indeed, as follows from

$$z(r) = \frac{r^6}{2 \int_0^r dr' \frac{r'^3}{(gW(r'))^2}}, \quad (3.19)$$

for a limited¹¹ superpotential, $W_0 \geq W \geq W_\infty > 0$, $z \approx 2(W_\infty r)^2$. As a consequence, $\xi(r) \rightarrow 1$ and the scalar equations (2.22) and (2.23) take the form:

$$\begin{aligned} \dot{\varphi}^\Lambda &= -3\partial^\Lambda \log W \equiv \beta^\Lambda, \\ &\approx \partial_\Sigma \beta^\Lambda (\varphi^\Sigma - \varphi_*^\Sigma), \end{aligned} \quad (3.20)$$

where we adopt the unified notation for the scalars φ^Λ , $\Lambda = 1, \dots, n_V + 4n_H$. φ_*^Σ represent the fixed-point values of the scalars for $r = \infty$. This tells us that any time the matrix

$$\mathcal{U}_\Sigma^\Lambda \equiv - \left. \frac{\partial \beta^\Lambda}{\partial \varphi^\Sigma} \right|_{\varphi_*} = \frac{3}{W} g^{\Lambda \Xi} \left. \frac{\partial^2 W}{\partial \varphi^\Sigma \partial \varphi^\Xi} \right|_{\varphi_*} \quad (3.21)$$

has positive eigenvalues, at least in the direction of the flow, the fixed-point is attractive for $r \rightarrow \infty$ and the flow can reach it. Thus, in the regime $r \approx \infty$ our problem coincides precisely with the domain wall one. This is not surprising because the configuration under study (2.5) can be seen as a spherical domain wall (the presence of an electric field is a necessary add-on for stabilizing it and solve the BPS equation): so for large r the two configuration become undistinguishable. Hence, all the BPS solutions obtained from compact gauging satisfying the properties derived in [7, 49] are asymptotically AdS_5 at the radial infinity.

When $r \approx 0$ the situation is totally different. As discussed in the previous section, now $\xi(r) \approx (W_0 r)^2$ (again, we assume that W_0 is finite). Then, in this region the different nature of a hypermultiplet scalar with respect to the vector multiplet one clearly emerge. Indeed, the former couples to the electric field and so it has a very distinct behavior in a charged configuration. From a practical point of view, as $\xi(r) \rightarrow 0$, the property of the fixed-point at $r = 0$ are not directly related to the eigenvalue of the Hessian (3.21). However, it is still possible to study the stability linearizing the flow equations (2.22) and (2.23). In order to explain clearly the procedure and to make evident the result, we sketch a generic model by considering two scalars ϕ and q representing the two different matter multiplets. After the

¹¹Moreover this implies that $2(W_\infty r)^2 < z < 2(W_0 r)^2$. The function z , which is related to the value of the electrostatic potential, interpolates monotonically between the two neutral AdS configurations.

linearization around a fixed-point, which without lose of generality we may fix at $\phi^* = q^* = 0$, the scalar equations reduced to

$$\dot{q} = \xi(r)(m\phi + Mq + O[\varphi]^2), \quad (3.22)$$

$$\dot{\phi} = -2\phi + mq + O[\varphi]^2, \quad (3.23)$$

where we used the fact that $U_x^y = 2\delta_x^y$ (cfr. (3.12)). Here m and M play the role of the Hessian blocks $-U_y^X$ ($-U_X^y$) and $-U_Y^X$ respectively. In order to integrate the above system we start by studying the equation (3.22).

We remind that $y = \ln r$ and $\dot{f} \equiv \frac{df}{dy} = r \frac{df}{dr}$. Regarding ϕ as a generic function of y , we can write:

$$q = m \exp \left[\frac{M}{2} \int dy \xi \right] \left(\int dy \xi \phi \exp \left[-\frac{M}{2} \int dy \xi \right] + c \right). \quad (3.24)$$

As we are interested in a solution that possibly reaches the fixed-point $q^* = 0$ for $y \rightarrow -\infty$ ($r \rightarrow 0$), we choose the integration constant accordingly:

$$q(y) = m \exp \left[\frac{M}{2} \int_{-\infty}^y dy' \xi(y') \right] \left(\int_{-\infty}^y dy' \xi(y') \phi(y') \exp \left[-\frac{M}{2} \int_{-\infty}^{y'} dy'' \xi(y'') \right] \right). \quad (3.25)$$

Here we need only the leading term in q that corresponds to the first contribution in ξ , as ξ goes to 0 at the fixed point. Thus, we get

$$q(y) = m \int_{-\infty}^y dy' \xi(y') \phi(y'). \quad (3.26)$$

We can substitute the above equation in the differential equation for ϕ :

$$\dot{\phi}(y) + 2\phi(y) - m^2 \int_{-\infty}^y dy' \xi(y') \phi(y') = 0. \quad (3.27)$$

This equation can be solved analytically in terms of Bessel functions. Indeed, by defining $\chi = \int_{-\infty}^y dy' \xi(y') \phi(y')$ and using that $\xi \propto e^{2y}$ around $r = 0$, (3.27) is equivalent to

$$\ddot{\chi} = m^2 W_0^2 e^{2y} \chi, \quad (3.28)$$

that is solved by

$$\chi = c_1 \text{BesselI}[0, |m|W_0 e^y] + c_2 \text{BesselK}[0, |m|W_0 e^y]. \quad (3.29)$$

It is easy to check that or χ is identically zero otherwise its limit $\lim_{y \rightarrow -\infty} \chi$ is always different from zero for any value of the constants c_1, c_2 as $\lim_{y \rightarrow -\infty} \text{BesselI}[0, |m|W_0 e^y] = 1$ and $\lim_{y \rightarrow -\infty} \text{BesselK}[0, |m|W_0 e^y] = \infty$. It follows that $\phi(y)$ diverges at the fixed-point that is always repulsive. The calculation proves what the intuition naively suggests: the presence of ξ that is going to zero at the fixed-point “cancels out” of the Hessian (3.21) all the entrances related to the hypermultiplet sector and only the repulsive contribution coming from vector

multiplets remains. Said in another way, from the vector multiplet point of view, due to the factor ξ , the hyperscalars can be treated as constant: thus we reduce effectively to the case when only vector multiplets are coupled.

It remains to check that the result we got from the toy-model (3.22-3.23) remains unchanged when we consider the full-fledged scalar equations. Indeed, this is the case. By first we observe that only the scalars whose derivatives receive contribution at the linear level from the mixing terms are affected by the presence of both the multiplets at the same time. Hence, only the flows in these directions have to be discussed as they are the only left over by the no-go theorem of the previous sections. This means that we can always choose a base in which we consider only the scalars $q^{\tilde{X}}, \phi^{\tilde{x}}$, with $\tilde{X}, \tilde{x} = 1, \dots, l \equiv \text{rank}[U_X^x]$, and $U_{\tilde{X}}^{\tilde{x}} = m_{(\tilde{x})} \delta_{\tilde{X}}^{\tilde{x}}$. Furthermore, as also the matrix $U_{\tilde{X}}^{\tilde{Y}}$ is symmetric, we can diagonalize it by a rotation on the $q^{\tilde{X}}, U_{\tilde{X}}^{\tilde{Y}} = M_{(\tilde{Y})} \delta_{\tilde{X}}^{\tilde{Y}}$, and at the same time keep $U_{\tilde{X}}^{\tilde{x}}$ in the diagonal form by antirotating the $\phi^{\tilde{x}}$. Hence, we end up with l copies of the system (3.22-3.23),

$$\dot{q}^i = \xi(r)(m_{(i)}\dot{\phi}^i + M_{(i)}q^i + O[\varphi]^2), \quad (3.30)$$

$$\dot{\phi}^i = -2\phi^i + m_{(i)}q^i + O[\varphi]^2, \quad (3.31)$$

where m and M are replaced by the eigenvalues $m_{(i)}$ and $M_{(i)}$. As a final remark, let us stress that our conclusions are not affected by non linear terms. These enter the game when the linear contribution around the fixed-point is zero. Indeed, this is obviously never the case when only vector multiplets are coupled. On the contrary, this is possible in the hypermultiplets sector. However, when the vector multiplets are not coupled the analysis of (3.18) remains unchanged as λ in any case should be divergent while it is not. Regarding the generic case, as the q 's can scale *at most*¹² as the ϕ 's, we can trust the Taylor expansion of the scalar derivatives, in particular of the $\dot{\phi}$'s. Thus, the only relevant mixing term that can appear in the $\dot{\phi}$'s (and as a consequence in the \dot{q} 's) is the linear one discussed above. If this is absent, we can directly conclude that the ϕ 's have a repulsive behavior as $\dot{\phi} \approx -2\phi$.

Therefore, we conclude that the hypothesis of a superior bound W_0 for the superpotential is contradicted by the absence of an attractive fixed-point for $r \rightarrow 0$. This proves that the existence of regular electrostatic and spherically symmetric BPS soliton is ruled out for *any* model of $N = 2, d = 5$ gauged supergravity with matter coupling.

4 Conclusion

Any supersymmetric solution different from the vacuum ends with a singularity in the interior of the spacetime. In practice all the BPS solutions that preserve strictly half of the supersymmetry behave qualitatively like a superstar [23] around $r \approx 0$, while at the radial infinity they may be asymptotical to (maximal supersymmetric) AdS_5 or they may display a runaway behavior as the analytical solution presented in [31]. This common behavior at

¹²This fact can be proven by using the l'Hôpital theorem in order to calculate the limit of the ratio $\frac{q}{\phi}$, $\lim_{\phi \rightarrow 0} \frac{q}{\phi} = \lim_{\phi \rightarrow 0} \frac{\dot{q}}{\dot{\phi}}$. This is essentially a consequence of $\xi \rightarrow 0$.

the origin can be interpreted as a sort of attractor mechanism. Indeed only the ultraviolet (UV) seems sensible to the details of the supergravity model.

The finding of this universal singular behavior at $r = 0$ has various consequences and suggests new questions. First of all, for an asymptotically AdS_5 solution it maps via holography in a universal behavior of RG flow of the dual theory, supposed it exists. Indeed, by adapting the argument of [51] (see also [52]), we can construct a c-function $C(r)$ that at large r (UV) will correspond to the central charge of the conformal theory on $R \times S^3$ [53]. Now, if we trust the c-theorem even if the solutions is not reaching any horizon in the infrared, we conclude that the RG flow always ends with zero central charge, as $C(r)$ is proportional (as in the domain wall case) to $1/W^3$.

A related issue is given by the interpretation of the asymptotically AdS_5 BPS solutions in terms of type IIB string theory. In the work of [36] it was shown that the superstars correspond to a singular limit of regular AdS bubbles. In spite of the fact that generically IIB solutions can not be consistently truncated to the $N = 2$, $d = 5$ solutions, it is very intriguing to conjecture that behind this universal singular behavior a new relation between pentadimensional and ten-dimensional configurations may be hidden. If this is the case, the corresponding ten-dimensional regular solutions should preserve at least 1/8 of the 32 supercharges. As the BPS equations for these class of configurations have been recently obtained by [54, 55] (see also [56]), it should be possible to check whether the interpretation of the superstars as 1/2 BPS AdS bubbles can be generalized and extended, at least in some special cases.

It would be very interesting to see whether our no-theorem extend to fake supergravity theories. Indeed, our result does not depend on the precise details of the model but on the relation between the superpotential W and the electric field F_{rt} , exemplified in the function z , as this relation determines the properties of the scalar flow. These last properties should only depend on the supersymmetric invariance at the linear order [38]. The only point where the full-fledge supersymmetric invariance seems to play a crucial role is in the determination of the special geometry and, as a consequence, of the Hessian of the superpotential. For this reason, we may naively suppose that also fake BPS electrostatic and spherically symmetric black holes are ruled out while solitons could exist. In order to discuss this issue, the fake supergravity formalism¹³ of [38] must be extended to include charged matter. An indication of how this can be done should come from the relation between $N = 2$, $d = 5$ supergravity and fake theory discussed in [15]. This issue is currently under investigation.

Another question raised by our work, is what is going to change if we consider configurations with the same symmetries but in presence of non-abelian gauging. In particular would be very interesting to consider the existence of black-holes. Due to the technical difficulties that such a study implies, the use of the classification achieved in [14] seems crucial.

¹³A different Fake supergravity formalism has been developed in [57, 58, 59] with the aim of studying the attractor mechanism of extreme non BPS black hole in $N = 2$ supergravity in four and five dimensions.

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A Spherically symmetric configurations $N = 2$ $d = 5$ gauged supergravity

A.1 Five-dimensional $N = 2$ gauged supergravity

We start by recalling some of the most important features of five-dimensional, $\mathcal{N} = 2$ gauged supergravity theories. Further technical details can be found in the original references [60, 61, 62, 47, 48].

The matter multiplets that can be coupled to $5D, \mathcal{N} = 2$ supergravity are vector, tensor and hypermultiplets: as we are interested to abelian gauging we end up with vector multiplets and hypermultiplets only.

The (n_V) scalar fields of n_V vector multiplets parameterize a “very special” real manifold \mathcal{M}_{VS} , i.e., an (n_V) -dimensional hypersurface of an auxiliary $(n_V + 1)$ -dimensional space spanned by coordinates h^I ($I = 0, 1, \dots, n_V + 1$) :

$$\mathcal{M}_{\text{VS}} = \{h^I \in \mathbb{R}^{(n_V+1)} : C_{IJK} h^I h^J h^K = 1\}, \quad (\text{A.1})$$

where the constants C_{IJK} appear in a Chern-Simons-type coupling of the Lagrangian. On \mathcal{M}_{VS} , the embedding coordinates h^I become functions of the physical scalar fields, ϕ^x ($x = 1, \dots, n_V$). The metric on the very special manifold is determined via the equations

$$\begin{aligned} g_{xy} &= h_x^I h_{yI}, & h_x^I &\equiv -\sqrt{\frac{3}{2}} \partial_x h^I, & h_I &\equiv C_{IJK} h^J h^K, & h_{Ix} &\equiv \sqrt{\frac{3}{2}} \partial_x h_I, \\ h^I h_J + h_x^I g^{xy} h_{yJ} &= \delta_J^I, & h^J h_I &= 1, & h^I h_{Ix} &= 0. \end{aligned} \quad (\text{A.2})$$

Some useful relations are

$$\begin{aligned} h_{Ix;y} &= \sqrt{\frac{2}{3}} (h_I g_{xy} + T_{xyz} h_I^z), \\ h^I{}_{x;y} &= -\sqrt{\frac{2}{3}} (h^I g_{xy} + T_{xyz} h^{Iz}), \end{aligned} \quad (\text{A.3})$$

where ‘;’ is a covariant derivative using the Christoffel connection calculated from the metric g_{xy} , with

$$T_{xyz} \equiv C_{IJK} h_x^I h_y^J h_z^K. \quad (\text{A.4})$$

Furthermore, we can define the auxiliary metric a_{IJ} of tangent space (h^I, h_x^I) , which determine the kinetic term of the gauge field in supergravity Lagrangian, as $a_{IJ} = h_I h_J + h_I^x h_{Jx}$.

The scalars q^X ($X = 1, \dots, 4n_H$) of n_H hypermultiplets, on the other hand, take their values in a quaternionic-Kähler manifold \mathcal{M}_Q [63], i.e., a manifold of real dimension $4n_H$ with holonomy group contained in $SU(2) \times USp(2n_H)$. We denote the vielbein on this manifold by f_X^{iA} , where $i = 1, 2$ and $A = 1, \dots, 2n_H$ refer to an adapted $SU(2) \times USp(2n_H)$ decomposition of the tangent space. The hypercomplex structure is (-2) times the curvature of the $SU(2)$ part of the holonomy group¹⁴, denoted as R^{rZX} ($r = 1, 2, 3$), so that the quaternionic identity reads

$$R_{XY}^r R^{sYZ} = -\frac{1}{4} \delta^{rs} \delta_X^Z - \frac{1}{2} \varepsilon^{rst} R_X^t{}^Z. \quad (\text{A.5})$$

Besides these scalar fields, the bosonic sector of the matter multiplets also contains n_V vector fields from the n_V vector multiplets. Including the graviphoton, we thus have a total of $(n_V + 1)$ vector fields, A_μ^I ($I = 0, 1, \dots, n_V$), which can be used to gauge up to $(n_V + 1)$ isometries of the quaternionic-Kähler manifold \mathcal{M}_Q (provided such isometries exist). As we restrict to abelian gauging only, there is no action on the “very special” real manifold \mathcal{M}_{VS} .

The quaternionic Killing vectors $K_I^X(q)$ that generate the isometries on \mathcal{M}_Q can be expressed in terms of the derivatives of $SU(2)$ triplets of Killing prepotentials $P_I^r(q)$ ($r = 1, 2, 3$) via

$$D_X P_I^r = R_{XY}^r K_I^Y, \quad \Leftrightarrow \quad \begin{cases} K_I^Y = -\frac{4}{3} R^{rYX} D_X P_I^r \\ D_X P_I^r = -\varepsilon^{rst} R_{XY}^s D^Y P_I^t, \end{cases} \quad (\text{A.6})$$

where D_X denotes the $SU(2)$ covariant derivative, which contains an $SU(2)$ connection ω_X^r with curvature R_{XY}^r :

$$D_X P^r = \partial_X P^r + 2 \varepsilon^{rst} \omega_X^s P^t, \quad R_{XY}^r = 2 \partial_{[X} \omega_{Y]}^r + 2 \varepsilon^{rst} \omega_X^s \omega_Y^t. \quad (\text{A.7})$$

The prepotentials satisfy the constraint

$$\frac{1}{2} R_{XY}^r K_I^X K_J^Y - \varepsilon^{rst} P_I^s P_J^t + \frac{1}{2} f_{IJ}^K P_K^r = 0, \quad (\text{A.8})$$

where f_{IJ}^K are the structure constants of the gauge group.

In the following, we will frequently switch between the above vector notation for $SU(2)$ -valued quantities such as P_I^r , and the usual (2×2) matrix notation,

$$P_{Ii}{}^j \equiv i \sigma_{ri}{}^j P_I^r. \quad (\text{A.9})$$

The general Lagrangian of the gauged supergravity theory under consideration is

$$\begin{aligned} \mathcal{L}_{\text{BOS}} = & \frac{1}{2} e \left\{ R - \frac{1}{2} a_{IJ} F_{\mu\nu}^I F^{J\mu\nu} - g_{XY} D_\mu q^X D^\mu q^Y - g_{xy} \partial_\mu \phi^x \partial^\mu \phi^y - 2g^2 \mathcal{V}(q, \phi) \right\} \\ & + \frac{1}{6\sqrt{6}} \varepsilon^{\mu\nu\rho\sigma\tau} C_{IJK} F_{\mu\nu}^I F_{\rho\sigma}^J A_\tau^K, \end{aligned} \quad (\text{A.10})$$

¹⁴In fact, the proportionality factor includes the Planck mass and the metric, which are implicit here.

where

$$D_\mu q^X = \partial_\mu q^X + g A_\mu^I K_I^X(q).$$

Then the variations of the fermions for abelian gauge symmetry $U(1)^{n_V+1}$ reduce to:
for the gravitini

$$\begin{aligned} \delta_\epsilon \psi_{\mu i} &= \partial_\mu \epsilon_i + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon_i - \partial_\mu q^X p_{Xi}{}^j \epsilon_j + g A_\mu^I P_{Ii}{}^j \epsilon_j \\ &+ \frac{i}{4\sqrt{6}} (\gamma_{\mu\nu\rho} - 4g_{\mu\nu} \gamma_\rho) h_I F^{I\nu\rho} \epsilon_i - \frac{i}{\sqrt{6}} g h^I P_{Ii}{}^j \gamma_\mu \epsilon_j = 0; \end{aligned} \quad (\text{A.11})$$

for the gaugini

$$\delta_\epsilon \lambda_i^X = -\frac{i}{2} \gamma^a \partial_a \phi^X \epsilon_i - \frac{i}{2} \gamma^a g A_a^I K_I^X + \frac{1}{4} h_I^X \gamma^{ab} F_{ab}^I \epsilon_i - g h^{XI} P_{Ii}{}^j \epsilon_j = 0 \quad (\text{A.12})$$

for the hyperini

$$\delta_\epsilon \zeta^A = \left[-\frac{i}{2} \gamma^a \partial_a q^X - \frac{i}{2} \gamma^a g A_a^I K_I^X + g h^I \frac{\sqrt{6}}{4} K_I^X \right] f_{Xi}^A \epsilon^i = 0 \quad (\text{A.13})$$

We are now ready to discuss the BPS equations for electrostatic and spherically symmetric configuration.

A.2 BPS equation for spherically symmetric configurations

Now, we review the main steps of derivation of the BPS equations as originally done [31, 32]. First of all we select the form of the ansatz. We are looking for electrostatic spherically symmetric solutions that preserve half of the $N = 2$ supersymmetries. We choose metric, which is $SO(4)$ -symmetric, as

$$ds^2 = -e^{2v} dt^2 + e^{2w} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\xi^2). \quad (\text{A.14})$$

with the function v and w , as well as the scalars, that only depend on the spacetime coordinate r . The same can be achieved for the gauge fields by fixing the gauge in order to have only the A_t^I component different from zero. Thus we can write

$$A_t^I = \sqrt{\frac{3}{2}} a^I e^v, \quad (\text{A.15})$$

where a^I are generic functions of r . However, as a vector, it can be decomposed on the flow as $a^I = a h^I + l^x h_x^I$. It is shown in [32] that the choice $l^x = 0$ is always compatible with the supersymmetry preservation. Furthermore, the presence of a non trivial l^x cannot affect the result of section 3, as the scalar fields and the metric are independent of l^x . For this reason, we consider only the case $l^x = 0$.

The next step is to substitute the above ansatz in the supersymmetry transformations in order to find a covariantly constant spinor ϵ . The BPS equations will arise as consistency conditions. A smart procedure is to consider first the matter transformations. Due to the ansatz, the gaugini (A.12) and hyperini (A.13) variations become respectively:

$$\begin{aligned} \delta_\epsilon \lambda_i^x &= \left[-i\phi'^x e^{-w} \gamma_1 \delta_i^j - 2igh^{Ix} P_I^s (\sigma_s)_i^j \right. \\ &\quad \left. + \sqrt{\frac{3}{2}} e^{-w} h_I^x \left(v' a^I + a^{I'} \right) \gamma_{01} \delta_i^j \right] \epsilon_j = 0 \end{aligned} \quad (\text{A.16})$$

and

$$\delta_\epsilon \zeta^A = f_{iX}^A \left[-iq'^X e^{-w} \gamma_1 + i\sqrt{\frac{3}{2}} ga^I K_I^X \gamma_0 + \sqrt{\frac{3}{2}} gh^I K_I^X \right] \epsilon^i = 0, \quad (\text{A.17})$$

where we use the notation $\chi' \equiv \partial_r \chi$. The projector content of the above equation can be written as

$$(-i\Lambda \gamma_0 \delta_l^k + f^r (\sigma_r)_l^k \gamma_1) \epsilon_k = \epsilon_l, \quad (\text{A.18})$$

where the consistency imposes $\Lambda^2 + f^r f_r = 1$.

By the use of the identities of the quaternionic manifold, the hyperini equation turns out to be equivalent (see also [12, 22]) to

$$f^r = \pm \sqrt{1 - \Lambda^2} Q^r \quad (\text{A.19})$$

$$q'^Z = \pm 3ge^w \sqrt{1 - \Lambda^2} \partial^Z W, \quad (\text{A.20})$$

$$\sqrt{1 - \Lambda^2} = \mp ge^w r W. \quad (\text{A.21})$$

As the projector condition coming from the hyperini equation must coincide with the one obtained from the gaugini, we get the constraint $\partial_x Q^r = 0$ and the BPS equation for the ϕ 's:

$$\phi'^x = \pm 3ge^w \frac{1}{\sqrt{1 - \Lambda^2}} \partial^x W. \quad (\text{A.22})$$

Furthermore, by consistency the electrostatic potential is fixed to be

$$A_t^I = \sqrt{\frac{3}{2}} \Lambda e^v h^I, \quad (\text{A.23})$$

as $a^I = \Lambda h^I$. Finally, the integrability condition of the gravitini turns out to be compatible with the above equations if

$$e^v = \frac{r}{r_0 \sqrt{1 - \Lambda^2}}. \quad (\text{A.24})$$

Let us summarize the results. The preservation of half of the supersymmetry, associated to the projector condition

$$\left(-i\Lambda \gamma_0 \delta_i^j \pm \sqrt{1 - \Lambda^2} Q^r (\sigma_r)_i^j \gamma_1 \right) \epsilon_j = \epsilon_i, \quad (\text{A.25})$$

is equivalent for an electrostatic and spherically symmetric solution to the BPS equations

$$\partial_x Q^r = 0, \quad (\text{A.26})$$

$$1 = \Lambda^2 e^{-2w} \left[1 + \frac{r}{2} \left(v' + \frac{\Lambda'}{\Lambda} \right) \right]^2, \quad (\text{A.27})$$

$$e^v = \frac{r}{r_0 \sqrt{1 - \Lambda^2}}, \quad (\text{A.28})$$

$$\sqrt{1 - \Lambda^2} = \mp g e^w r W, \quad (\text{A.29})$$

$$q'^Z = \pm 3 g e^w \sqrt{1 - \Lambda^2} \partial^Z W, \quad (\text{A.30})$$

$$\phi'^x = \pm 3 g e^w \frac{1}{\sqrt{1 - \Lambda^2}} \partial^x W. \quad (\text{A.31})$$

As explicitly shown in [32], the above equations are sufficient to ensure the equations of motion.

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